



Delegating climate policy to a supranational authority: a theoretical assessment[☆]



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ABSTRACT

This paper studies the delegation of climate policy to a supranational environmental authority. We develop a simple model of a world consisting of a large number of countries, which derive utility from energy consumption. Countries suffer from global warming and local air pollution, both caused by the combustion of fossil fuels, and decide individually on investments in clean technologies for energy production. A supranational environmental authority decides for each country on the maximally permitted amount of greenhouse gas emissions. We demonstrate that the authority faces a dynamic inconsistency problem that leads to welfare losses, but these losses can be kept small if the authority is endowed with an optimally designed mandate. The optimal mandate penalizes the cost of local air pollution very heavily relative to the cost of global warming. However, delegation of climate policy faces a further difficulty, as countries have a recurrent incentive to change the authority's mandate over time.

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1. Introduction

Many scientists today consider global warming the biggest threat humanity has ever faced. It will lead to severe disruptions of everyday life around the globe and it is an extremely difficult problem to tackle. The difficulties arise primarily for two reasons. First, global warming is due to strong negative external effects of economic activity (in particular energy production and transportation) which complicates its solution by simple market mechanisms.¹ Second, it is a phenomenon that can only be addressed on a global basis. To be effective, measures against global warming have to be implemented on a broad international scale, which requires difficult and protracted negotiations between many heterogeneous countries with very diverse goals; (see, e.g., Barrett, 2003 or Nordhaus, 2013). In the past, such negotiations often had limited success

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¹ Stern (2007, p. 1) refers to global warming as “the greatest example of market failure we have ever seen”.

or failed altogether.² Against this background, there have been several proposals to create a supranational environmental authority (SEA) with the explicit mandate to fight global warming, and to delegate decision power over certain climate-relevant policies to this authority. Their proponents include academic researchers (e.g., Esty, 1994, Whalley and Zissimos, 2002, Helm et al., 2003, Biermann and Bauer, 2005, Barnes et al., 2008, Grosjean et al., 2016), economic policy advisors (e.g., German Council of Economic Experts, 2013) and political commentators (e.g., The Politic, 2014).

In the present paper we take these proposals as a starting point and assess the desirability of delegating climate policy to a supranational authority. We develop a stylized model of a world consisting of a large number of countries, which derive utility from energy consumption but suffer from the negative consequences of emissions generated by fossil fuel combustion. These consequences take two forms. First, emissions of carbon dioxide (CO₂) and other greenhouse gases contribute to global warming, and hence cause global costs that are not internalized by the individual countries. Second, emissions of compounds such as sulfur dioxide (SO₂) or black carbon (PM_{2.5}) cause local air pollution, and hence local costs. All countries can mitigate the negative side-effects of their energy consumption by investing into clean technologies for energy production. Each country decides individually on its investment into clean technologies, but its decision over emissions (climate policy) is delegated to an SEA, which imposes emission quotas for each country.³ We abstract from potential difficulties concerning the countries' compliance with the quotas set by the SEA and the implementation of policies via appropriate instruments, such as tradable pollution permits. We also abstract from uncertainty about the true economic costs of local air pollution and global warming.⁴ Instead, our main focus is on the incentives and the dynamic behavior of the authority.

We first show that, even if all countries respect the SEA's policy prescriptions and the authority has global welfare as its mandate, the optimal climate policy is dynamically inconsistent. *Ex ante*, the SEA finds it optimal to set tight emission quotas in order to induce countries to invest into clean technologies. *Ex post*, after the investments are sunk, providing investment incentives is no longer important, and tight emission quotas are no longer optimal. Rather, the authority would like to relax emission quotas to allow for more energy consumption. Individual countries anticipate that the authority will eventually not find it worthwhile to enforce tight emission quotas, which reduces their incentive to invest into clean technologies already from the start. Discretionary behaviour of the authority therefore causes under-investment into clean technologies, over-pollution and, accordingly, a welfare loss relative to the situation when the authority has commitment power.

We then demonstrate that the dynamic inconsistency problem can be ameliorated by giving the SEA a mandate that differs from the maximization of global welfare. In particular, an SEA that is endowed with an optimally designed mandate can implement a discretionary policy with significantly smaller welfare loss than a benevolent SEA. The optimal mandate attaches a *higher* weight to the *local* cost of emissions (local air pollution) than the individual countries themselves, and a relatively low or even zero weight to the *global* costs (the damage due to global warming). The intuition behind this seemingly counter-intuitive result is as follows. An authority that is less willing to accept local air pollution has a stronger incentive to enforce tight emission quotas *ex post*, even for those countries that invest little into clean technologies. Moreover, its incentive to impose tight quotas is largely independent of the stock of greenhouse gases in the atmosphere. A mandate with a large weight on local air pollution costs thus provides partial commitment to the authority, which alleviates its time-inconsistency problem. Given a large weight on local air pollution, it is further optimal to put a relatively small weight on global warming costs, in order not to distort the optimal trade-off between clean and dirty energy consumption too much. The optimal mandate is thus of the spirit *act local, solve global*, a spirit that has recently been promoted by the IMF in the context of energy policy.⁵

Finally, we show that the optimal mandate of the authority is not state-invariant but depends on the initial stock of greenhouse gases in the atmosphere. The higher is this stock, the more *environmentalist* the mandate of the authority should be, i.e. the higher the optimal weights on the costs of emissions relative to the utility benefits of energy consumption. Intuitively, this is because an environmentalist authority provides stronger incentives for large initial investments in clean technologies, which ensures a fast reduction of greenhouse gases. As the stock of greenhouse gases is reduced over time, however, the optimal mandate becomes less and less environmentalist. This creates another dynamic inconsistency problem, this time on the side of the individual countries. Even if they can commit to set up an SEA and to respect its policy prescriptions, as we do assume throughout our analysis, they have a recurrent incentive to change the authority's mandate as the stock of greenhouse gases evolves over time.

Our framework is a modification of the model used by Harstad (2012), Harstad (2016), and Battaglini and Harstad (2016), which was developed to analyze international environmental agreements. We depart from this framework along three dimensions. First, as the purpose of our analysis is to study optimal delegation of climate policy, emission levels are not decided and negotiated by individual countries but imposed by an SEA. Second, we describe the world as consisting of many small countries rather than a finite number of (large) countries. This rules out strategic interactions between coun-

² The Copenhagen Summit in 2009 is often mentioned as a prime example of failure; see, e.g., the results of the Symposium on International Climate Negotiations summarized in Cramton et al. (2015). Barrett (1994) argues that international environmental agreements need to be self-enforcing and, therefore, they are not likely to be signed by many countries unless the global benefits of cooperation are small.

³ Note that an alternative interpretation of our model is that of an environmental protection agency (EPA) which has the power to impose emission standards on firms on a national level. Due to the global nature of global warming, however, we prefer to think of the authority as a supranational one.

⁴ Compliance, implementation, and uncertainty are clearly all very important in practice. Yet, addressing these issues is beyond the scope of the present theoretical analysis and left for future research.

⁵ See, e.g., Parry et al. (2013) or Clements and Gaspar (2015).

tries and allows us to focus more clearly on the properties of the authority's optimal policy problem. Third, we introduce local costs of fossil fuel combustion. As we have argued before, accounting for local costs is relevant because most economic activities that generate greenhouse gas emissions typically also lead to other emissions that have local costs, e.g. sulfur dioxide or black carbon (soot).⁶ These local costs are empirically sizeable. According to Parry et al. (2013, p.1), “outdoor air pollution, primarily from fossil fuel combustion, causes more than 3 million premature deaths a year worldwide, costing about 1 percent of GDP for the United States and almost 4 percent for China.”

This paper contributes to a large literature on dynamic inconsistency and coordination of environmental policy. Gersbach and Glazer (1999) point out that dynamic inconsistency creates a hold-up problem for firms, which can be solved by introducing a market for emission permits. Petrakis and Xepapadeas (2003) study the location decisions of a monopolistic firm that is subject to emission taxes in its home country. Among other things they show that the government's commitment to an emission tax is not necessarily welfare improving. D'Amato and Dijkstra (2015) analyze firms' investment decisions in clean technologies under asymmetric information about costs of abatement with the new technology. They show that a regulator who sets emission taxes or emission permits cannot implement the first-best solution under commitment, but that this can be done with a time consistent discretionary policy. Climate cooperation has been studied by Barrett (1994) in the context of self-enforcing international environmental agreements and by Gersbach and Winkler (2011) in the context of international emission permit markets with refunding. Finally, Helm and Schmidt (2015) develop a model of climate cooperation with endogenous research and development investments where countries protect their international competitiveness via border carbon adjustments. All of these papers, however, use a modelling framework that is very different from the one in the present paper, and they do not discuss the question of optimal delegation at all.

Optimal delegation is addressed extensively in the literature on monetary policy, where dynamic inconsistency problems can be extenuated by augmenting the central bank's mandate by strong inflation aversion, as in Rogoff (1985), or by a desire for policy inertia, as in Woodford (2003).⁷ In a general context, it has recently been analyzed in Alonso and Matouschek (2008), Amador and Bagwell (2013), and Krähmer and Kováč (2016), among others. To the best of our knowledge, the so far only formal application of optimal delegation in environmental policy is provided by Helm et al. (2004). These authors study emission taxation of firms in a very simple, essentially static model and find that delegation to an “environmentalist policy maker” is beneficial to the society. The environmentalist policy maker attaches higher weight than the society to the costs of pollution. Due to its simple structure, however, the model in Helm et al. (2004) does not capture the stock externality of greenhouse gas emissions at all, nor does it distinguish between local and global costs. The present paper, in contrast, analyzes the design of an SEA in a fully dynamic framework featuring the accumulation of greenhouse gases in the atmosphere and the resulting negative stock externality. McCallum (1995) and Jensen (1997) note that the delegation of monetary policy to an independent central bank may not be able to resolve the dynamic inconsistency problem but only relocates it from the bank to the government which decides on the mandate of the bank.⁸ We observe a similar property in our model and show that the countries may want to change the mandate of the SEA in the course of time.

The rest of this paper is organized as follows. Section 2 describes the economic and ecological environment. Section 3 studies two extreme benchmarks: business as usual (BAU) and the first-best solution.⁹ Section 4 analyzes delegation of emission decisions to an SEA which pursues the goal of maximizing aggregate welfare in the world, while Section 5 analyzes the optimal delegation problem. Section 6 discusses the dynamic inconsistency problem of the countries that results from the state-dependence of the optimal mandate given to the SEA. Finally, Section 7 concludes. All proofs are presented in the appendix at the end of the paper.

2. The economic environment

We consider a world consisting of a unit interval of infinitesimally small countries $i \in [0, 1]$. Energy can be produced either by a clean technology or by a dirty one. Only the latter generates greenhouse gas emissions. We denote by $R_{i,t}$ the stock of clean technology available in country i at time t and by $g_{i,t}$ the amount of emissions generated by this country in period t . We choose units of measurement in such a way that $R_{i,t} + g_{i,t}$ denotes the total amount of energy available to country i in period t .¹⁰ Furthermore, we denote by $r_{i,t}$ country i 's investment in the clean technology in period t . This implies that

$$R_{i,t} = r_{i,t} + \rho R_{i,t-1}. \quad (1)$$

⁶ Note that CO₂ itself has no negative effects at the location where it is emitted but it stays in the atmosphere for very long and causes global warming.

⁷ Two prominent actual implementations of these ideas are the inflation targeting mandate assigned to the Swedish Riksbank in 1993, and the European Central Bank's mandate, which assigns it the primary objective of maintaining price stability and a secondary objective of supporting the general economic policies of the Union.

⁸ This critique of the optimal delegation literature has been somehow mitigated by Driffill and Rotondi (2006), though.

⁹ BAU describes the situation in which all individual countries pursue their own interests without any coordination among them. The first-best solution is the allocation that would be implemented by a social planner who makes all investment and emission decisions for all countries and seeks to maximize aggregate welfare in the world.

¹⁰ This assumption means that we define one unit of the stock of clean technology as the capacity to generate as much energy as the dirty technology can generate with one unit of emissions. It also entails that the services of the clean and dirty technology are perfect substitutes.

where $1 - \rho \in (0, 1)$ is the rate of depreciation of the technology stock. An initial value $R_{i,-1}$ is exogenously given for every country $i \in [0, 1]$.

The total stock of greenhouse gases in the atmosphere at time t is denoted by G_t . It is assumed that

$$G_t = \int_0^1 g_{i,t} di + \gamma G_{t-1}, \quad (2)$$

where $1 - \gamma \in (0, 1)$ is the natural decay rate of greenhouse gases. The initial stock G_{-1} is exogenously given.

The utility derived by country i during period t is measured by

$$U(G_t, R_{i,t}, g_{i,t}, r_{i,t}) = B(R_{i,t} + g_{i,t}) - C(g_{i,t}) - D(G_t) - kr_{i,t},$$

where $B(R_{i,t} + g_{i,t})$ is the benefit derived from energy consumption, $C(g_{i,t})$ measures the local cost of energy production by the dirty technology in country i , $D(G_t)$ denotes the cost generated by the (global) stock of greenhouse gases, and k is the constant unit cost of investment. The local cost $C(g_{i,t})$ comprises the actual production cost of energy (including, e.g., the cost of fossil fuels) and the cost of local air pollution. Note that all countries have the same utility function but that the countries can differ from each other with respect to their initial technology levels. We assume that $B : \mathbb{R} \mapsto \mathbb{R}$ is concave and smooth, that $C : \mathbb{R} \mapsto \mathbb{R}$ and $D : \mathbb{R} \mapsto \mathbb{R}$ are strictly convex and smooth, and that k is a positive constant. Denoting the common time-preference factor of all countries by $\beta \in (0, 1)$, we can write total discounted welfare derived by country i as

$$\sum_{t=0}^{+\infty} \beta^t U(G_t, R_{i,t}, g_{i,t}, r_{i,t}) \quad (3)$$

and aggregate world-welfare as

$$\sum_{t=0}^{+\infty} \beta^t \int_0^1 U(G_t, R_{i,t}, g_{i,t}, r_{i,t}) di. \quad (4)$$

The setup described above shares with that used by Harstad (2012) and Harstad (2016) the additive separability of the utility function and the linear investment cost. These properties ensure high analytical tractability. However, there are also important differences between our model and that of Harstad (2012) and Harstad (2016). In particular, we assume a continuum of countries and that these countries face costs of local emissions (described by the function C). As a consequence and contrary to the framework studied by Harstad (2012) and Harstad (2016), in our model there is no strategic interaction between the countries because each one is infinitesimally small. If the global cost of global warming were the only negative consequence of emissions, all countries would like to increase their emissions and, consequently, their benefits from energy consumption without bound, because these costs are external to the individual countries. The local cost of emitting greenhouse gases serves as a limiting factor to energy production and thereby ensures that an equilibrium exists. Moreover, it is a relevant aspect of real-world energy production. Finally, note that from Section 4 onwards, we will introduce a supranational environmental authority, which is also absent from Harstad (2012) and Harstad (2016). In that part of the paper it will be necessary to be more specific about the timing of events within every period. We shall follow Harstad (2012) and Harstad (2016) by assuming that the investment decisions on $r_{i,t}$ have to be made before the emission decisions $g_{i,t}$. Even if this timing assumption is not essential for the main insights of the paper, it greatly helps to keep the analysis tractable.

We neglect non-negativity constraints throughout the paper in order to avoid long and tedious technical derivations that would distract the readers from the main points of the paper. The variables for which non-negativity constraints are important are R_i and g_i . A non-negativity constraint on g_i automatically implies non-negativity of G . All the results from Sections 3 and 4.1 continue to hold in the presence of these constraints provided that reasonable parameter restrictions are imposed; (see Harstad, 2012, Section 5.2) for a very similar argument. Technical difficulties would arise, however, when we use Markovian strategies from Section 4.2 onwards. Imposing non-negativity constraints would require from us to work with piecewise linear rather than linear strategies, which would tremendously complicate the analysis. We are convinced, however, that the main mechanisms driving all our results do not hinge upon missing non-negativity constraints.

3. Benchmarks

In this section we describe two benchmarks that will be useful for assessing the value of commitment and delegation in the main part of the paper. The first benchmark describes business as usual, a situation in which all countries pursue their own interests and there is no coordination among them whatsoever. For the second benchmark we assume that there exists a social planner who chooses allocations in such a way that aggregate welfare is maximized. These two benchmarks correspond to no internalisation of the environmental externality and to full internalisation, respectively.

3.1. Business as usual

Suppose that all countries act separately in their own interest. The countries do neither coordinate their actions nor is there any institution that facilitates such a coordination. Since every country is infinitesimally small, it disregards its influence on the global stock of greenhouse gases. In other words, it neglects (2) and treats the stocks of greenhouse

gases $(G_t)_{t=0}^{+\infty}$ as exogenous to its own decision problem. We refer to the resulting equilibrium as business as usual (BAU-equilibrium).

Definition 1. A BAU-equilibrium consists of a global sequence $(G_t)_{t=0}^{+\infty}$ and individual sequences $\{(R_{i,t}, g_{i,t}, r_{i,t})_{t=0}^{+\infty} \mid i \in [0, 1]\}$ such that the following two conditions are satisfied: (i) Given $(G_t)_{t=0}^{+\infty}$ it holds for all $i \in [0, 1]$ that $(R_{i,t}, g_{i,t}, r_{i,t})_{t=0}^{+\infty}$ maximizes (3) subject to (1). (ii) Eq. (2) holds for all $t \geq 0$.

Lemma 1. The unique BAU-equilibrium is given by

$$g_{i,t} = g^{\text{BAU}}, \quad (5)$$

$$R_{i,t} = R^{\text{BAU}}, \quad (6)$$

$$G_t = \frac{g^{\text{BAU}}}{1-\gamma} + \left(G_{-1} - \frac{g^{\text{BAU}}}{1-\gamma}\right)\gamma^{t+1} \quad (7)$$

for all $i \in [0, 1]$ and all $t \geq 0$, where g^{BAU} and R^{BAU} are uniquely determined by

$$B'(R^{\text{BAU}} + g^{\text{BAU}}) = C'(g^{\text{BAU}}) = (1 - \beta\rho)k.$$

All formal proofs are relegated to the appendix. The above lemma shows that in the BAU-equilibrium all countries, irrespective of their initial technology levels, choose the same emission rates and technology levels from period $t = 0$ onwards.¹¹ In other words, the emission rates as well as the technology levels attain their steady state values g^{BAU} and R^{BAU} immediately from period 0 onwards. The global stock of greenhouse gases, however, evolves according to Eq. (7), which implies that it converges only asymptotically to the steady state value G^{BAU} determined by

$$C'((1-\gamma)G^{\text{BAU}}) = (1 - \beta\rho)k. \quad (8)$$

None of the BAU-equilibrium conditions involves the global cost function D . This is not surprising because every country takes the greenhouse gas levels $(G_t)_{t=0}^{+\infty}$ as given so that the cost of global warming, $D(G_t)$, is an additively separated term in the utility function which cannot be affected by any country alone.

3.2. First-best solution

Now suppose that a social planner makes the investment and emission decisions for all countries in order to maximize world welfare as given by (4). We call the resulting solution the first-best solution.

Definition 2. The first-best solution consists of a global sequence $(G_t)_{t=0}^{+\infty}$ and individual sequences $\{(R_{i,t}, g_{i,t}, r_{i,t})_{t=0}^{+\infty} \mid i \in [0, 1]\}$ which together maximize (4) subject to (1) and (2).

Lemma 2. The first-best solution is characterized by Eqs. (1) and (2) along with the first-order conditions

$$g_{i,t} = g_t, \quad (9)$$

$$R_{i,t} = R_t, \quad (10)$$

$$B'(R_t + g_t) = (1 - \beta\rho)k, \quad (11)$$

$$D'(G_t) = (1 - \beta\gamma)(1 - \beta\rho)k - C'(g_t) + \beta\gamma C'(g_{t+1}) \quad (12)$$

for all $i \in [0, 1]$ and all $t \geq 0$ as well as the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t [(1 - \beta\rho)k - C'(g_t)]G_t = 0. \quad (13)$$

Comparing the first-best solution with the BAU-equilibrium, we see that total energy consumption $R_{i,t} + g_{i,t}$ by every country $i \in [0, 1]$ and in any given period $t \geq 0$ is the same in both solutions. The only difference in the two solutions consists therefore in the way how this amount of energy is produced. In the BAU-equilibrium the technology level R^{BAU} is installed in period 0 and maintained forever, whereas in the first-best solution the sequences $(g_{i,t})_{t=0}^{+\infty}$ and $(R_{i,t})_{t=0}^{+\infty}$ are not

¹¹ This result rests on the linearity of the investment costs.

constant. Their evolution together with that of $(G_t)_{t=0}^{+\infty}$ is determined by Eqs. (2) and (11)–(13). The stock of greenhouse gases converges to G^{FB} defined by

$$C'((1-\gamma)G^{\text{FB}}) + \frac{D'(G^{\text{FB}})}{1-\beta\gamma} = (1-\beta\rho)k.$$

This equation says that the marginal emission damages (the local cost caused by the flow of emissions plus the present value of the global cost caused by the stock of greenhouse gases) must coincide with the marginal benefit of the emissions which, according to Eq. (11), is equal to $(1-\beta\rho)k$. Comparing the above equation to (8) and using strict convexity of C and D it follows that G^{FB} is smaller than G^{BAU} . This implies of course that the steady state emission rate g^{FB} in the first-best solution is smaller than g^{BAU} and, consequently, that the steady state technology level R^{FB} in the first-best solution must be larger than R^{BAU} . To summarize, full internalisation of the external effects of greenhouse gas emission does not change the benefit from energy consumption for any country but it speeds up the replacement of the dirty technology by the clean one.

3.3. The linear-quadratic case

Throughout the paper we will illustrate our results by considering a version of the model in which the benefit and cost functions are given by

$$B(y) = -\frac{(y-z)^2}{2}, \quad C(g) = \frac{cg^2}{2}, \quad D(G) = \frac{dG^2}{2}, \quad (14)$$

where c and d are positive parameters and z is a bliss point for energy consumption. These functional forms are borrowed from Harstad (2016), except for $C(g)$ which is absent in his analysis. The quadratic specifications allow us to compute the two benchmark solutions easily. For example, the steady state values in the BAU-equilibrium are given by

$$G^{\text{BAU}} = \frac{(1-\beta\rho)k}{(1-\gamma)c},$$

$$R^{\text{BAU}} = z - \frac{(1-\beta\rho)(1+c)k}{c},$$

whereas the corresponding values in the first-best solution are

$$G^{\text{FB}} = \frac{(1-\beta\gamma)(1-\beta\rho)k}{d + (1-\gamma)(1-\beta\gamma)c},$$

$$R^{\text{FB}} = z - \frac{[d + (1-\gamma)(1-\beta\gamma)(1+c)](1-\beta\rho)k}{d + (1-\gamma)(1-\beta\gamma)c}.$$

As has been mentioned before, it holds that $G^{\text{FB}} < G^{\text{BAU}}$ and $R^{\text{BAU}} < R^{\text{FB}}$.

4. Delegation to a benevolent authority

In this section we turn to the main investigation of this paper by introducing a supranational environmental authority (SEA), to which all countries $i \in [0, 1]$ delegate their decisions on the emission levels $(g_{i,t})_{t=0}^{+\infty}$. The countries keep control over their investments $(r_{i,t})_{t=0}^{+\infty}$ and seek to maximize their individual welfare given in (3).¹² We distinguish between the case where the SEA has commitment power so that it can announce and implement the entire family of sequences $\{(g_{i,t})_{t=0}^{+\infty} \mid i \in [0, 1]\}$ at the outset of period 0, and the case where it lacks such commitment power. In both cases we assume that the SEA is benevolent in the sense that it seeks to maximize aggregate welfare as given in (4).¹³

4.1. Commitment

In this subsection we assume that the SEA has commitment power and announces the emission levels for all countries and all periods at the outset of period 0. The countries act as followers. We start by determining the best response of an arbitrary country $i \in [0, 1]$ to the fixed family of sequences $\{(g_{j,t})_{t=0}^{+\infty} \mid j \in [0, 1]\}$.

Lemma 3. Let $(G_t)_{t=0}^{+\infty}$ and $\{(g_{j,t})_{t=0}^{+\infty} \mid j \in [0, 1]\}$ be given sequences such that condition (2) holds. For every $i \in [0, 1]$, country i 's best response is characterized by

$$B'(R_{i,t} + g_{i,t}) = (1-\beta\rho)k \text{ for all } t \geq 0. \quad (15)$$

¹² For reasons of political sovereignty we believe that it is much less plausible to assume that the countries delegate also their investment decisions which, in our model economy, would result in the implementation of the first-best solution.

¹³ We shall drop the assumption of benevolence in Section 5.

Using the above characterization of best responses, we can define an equilibrium under commitment. We then demonstrate that such an equilibrium coincides with the first-best solution.

Definition 3. An equilibrium under commitment consists of a global sequence $(G_t)_{t=0}^{+\infty}$ and individual sequences $\{(R_{i,t}, g_{i,t}, r_{i,t})_{t=0}^{+\infty} \mid i \in [0, 1]\}$ which maximize (4) subject to (1), (2), and (15).

Lemma 4. An equilibrium under commitment coincides with the first-best solution.

The above result is due to the negative externality in our framework operating solely via the emissions and the resulting stock of greenhouse gases. In particular, in the case where the authority commits to any family of sequences of emissions $\{(g_{j,t})_{t=0}^{+\infty} \mid j \in [0, 1]\}$, the countries choose corresponding efficient investment levels. This is reflected by the coincidence of Eqs. (11) and (15). Accordingly, when the authority can commit to setting all emission levels at the socially optimal values, the individual countries face the right incentives to set their investments at the socially optimal levels, and the first-best solution can be achieved.

The problem with this solution, however, is that it is not dynamically consistent. In period 0, the SEA finds it optimal to announce relatively tight emission quotas $\{(g_{j,t})_{t=0}^{+\infty} \mid j \in [0, 1]\}$, knowing that these provide strong incentives for the countries to invest into clean technologies. But in any future period, once the investments are installed, providing investment incentives is no longer important for this period and the authority would like to relax the current quota to allow for more energy consumption. The authority thus faces a recurrent temptation to relax the quotas announced in period 0 as time evolves.¹⁴ If the SEA lacks full commitment power, it will therefore not stick to its announced policy. The underlying mechanism is familiar from Harstad (2012), where it leads to the observation that climate treaties are generally not renegotiation-proof as they specify emission levels that are less than what is optimal after the investments are sunk.¹⁵

4.2. Discretion

Consider now the situation where the SEA cannot make a binding commitment. To formalize this idea, we assume that the authority makes a consistent plan in the sense of Strotz (1955–1956).¹⁶ This means that the SEA consists of an infinite sequence of selves, one for each period. In every period t the current self of the SEA decides on $\{g_{i,t} \mid i \in [0, 1]\}$ only. But in doing this, it correctly anticipates how its choice of the emission rates affects the investment behavior of the individual countries and the future emission levels determined by later selves of the authority. Formally, the different selves of the authority play a game among each other. We solve this game for a stationary Markov-perfect Nash equilibrium.¹⁷

A stationary Markov-perfect equilibrium can be defined most easily in recursive form using recursive notation. Consider any given period t . Variables dated t will be written without a subscript, e.g., G , R_i , g_i , and r_i . Variables of the preceding period $t-1$ will be denoted by the subscript $-$, e.g., G_- , $R_{i,-}$, $g_{i,-}$, and $r_{i,-}$. At the start of a given period the aggregate states G_- and $\tilde{R}_- = \{R_{i,-} \mid i \in [0, 1]\}$ are known to all countries and to the SEA, where \tilde{R}_- is interpreted as the distribution of technology levels.¹⁸ As for the timing, we follow Harstad (2012) and assume that every time period is split into two stages. In the first stage all countries invest into clean technologies (investment stage) and in the second stage the authority sets the emission levels and the countries consume (emission stage). Note that the emission stage begins once the investment costs are sunk and that the SEA knows the countries' technology levels at the end of the investment stage, $\tilde{R} = \{R_i \mid i \in [0, 1]\}$, when it has to decide on the emission levels $\{g_i \mid i \in [0, 1]\}$.

We restrict attention to those stationary Markovian strategies \mathbf{g} of the SEA according to which the authority's choice of the emission level for country i depends only on the aggregate state variables G_- and \tilde{R} as well as on country i 's own technology level R_i . We write such a strategy in the form $g_i = \mathbf{g}(G_-, \tilde{R}, R_i)$. It is clear that the authority should be able to condition the emission level g_i for country i on the aggregate states G_- and \tilde{R} . We also want the SEA to be able to react to country i 's technology level R_i in order to punish countries which do not invest enough. On the other hand, we want to rule out that country i 's emission level g_i (chosen by the authority) depends directly on the choice variables of other countries (such as R_j for $j \neq i$) because this would introduce the possibility of strategic interaction between the countries (indirectly, i.e., via the SEA's policy function).

¹⁴ Due to our timing assumptions, which imply that emission quotas are decided after countries make their investment decisions, the authority faces a commitment problem even *within* a given time period: if it would announce quotas before investment decisions are made, these quotas would no longer be optimal once investments are sunk. Changing our timing assumptions such that emission quotas are decided simultaneously with investment would remove the *within period* commitment problem, but would not affect the commitment problem across time periods.

¹⁵ Note that the intuition for the dynamic inconsistency is also very similar to the case of capital income taxation treated by Chamley (1986) and Judd (1985). The government implements a low tax rate in order to induce the households to accumulate physical capital, but once the investment has taken place and physical capital is inelastically provided, the government has an incentive to tax it heavily in order to reduce distorting labor taxation.

¹⁶ This approach is often called the sophisticated solution according to the multiple-selves model; see, e.g., chapter 6 in Sorger (2015) for a recent textbook presentation.

¹⁷ Stationarity means that every self of the SEA uses the same strategy. Markov-perfection is ensured by allowing only strategies that depend solely on the payoff-relevant state variables and by using backward induction for defining the equilibrium.

¹⁸ By referring to G_- and \tilde{R}_- as aggregate states we indicate that no individual country can affect these variables by unilateral actions. In particular, country i can decide on its own technology level R_i , but it cannot alter the entire distribution \tilde{R} . It will turn out that the equilibrium strategies are independent of the distribution \tilde{R}_- and that the equilibrium value functions depend on \tilde{R}_- only via the aggregate (or, equivalently, the average) technology level $\int_0^1 R_{i,-} di$; see Lemma 5 below.

Before we can define an equilibrium under discretion, we need to explain some preliminary considerations. To begin with, we need to find out how the countries react to a given strategy \mathbf{g} of the SEA. Suppose therefore that all selves of the SEA play the strategy \mathbf{g} . It follows from (2) that the stock of greenhouse gases evolves according to

$$G = \mathcal{G}(G_-, \tilde{R}), \quad (16)$$

where

$$\mathcal{G}(G_-, \tilde{R}) = \int_0^1 \mathbf{g}(G_-, \tilde{R}, R_i) di + \gamma G_-. \quad (17)$$

Now suppose that we are at the beginning of period t and consider an arbitrary country i that has technology level $R_{i,t-1}$. Since country i is infinitesimally small, it takes $(G_s)_{s=t-1}^{+\infty}$ and $(\tilde{R}_s)_{s=t-1}^{+\infty}$ as given and maximizes

$$\begin{aligned} & \sum_{s=t}^{+\infty} \beta^{s-t} [B(R_{i,s} + \mathbf{g}(G_{s-1}, \tilde{R}_s, R_{i,s})) - C(\mathbf{g}(G_{s-1}, \tilde{R}_s, R_{i,s})) - D(G_s) - k(R_{i,s} - \rho R_{i,s-1})] \\ & = k\rho R_{i,t-1} + \sum_{s=t}^{+\infty} \beta^{s-t} [B(R_{i,s} + \mathbf{g}(G_{s-1}, \tilde{R}_s, R_{i,s})) - C(\mathbf{g}(G_{s-1}, \tilde{R}_s, R_{i,s})) - D(G_s) - (1 - \beta\rho)kR_{i,s}] \end{aligned}$$

by choosing $(R_{i,s})_{s=t}^{+\infty}$. This is equivalent to setting $R_{i,t} = \mathbf{R}(G_{t-1}, \tilde{R}_t)$, where

$$\mathbf{R}(G_-, \tilde{R}) = \operatorname{argmax}_x \{B(x + \mathbf{g}(G_-, \tilde{R}, x)) - C(\mathbf{g}(G_-, \tilde{R}, x)) - (1 - \beta\rho)kx \mid x \in \mathbb{R}\}. \quad (18)$$

We proceed under the assumption that the maximization problem in (18) has a unique solution, i.e., that \mathbf{R} is a single-valued function.¹⁹ This implies in particular that, independently of the form of the distribution \tilde{R}_- , all countries $i \in [0, 1]$ choose the same technology level $\mathbf{R}(G_-, \tilde{R})$ so that the distribution \tilde{R} is degenerate. This feature is due to the linear investment cost. The distribution \tilde{R} can of course depend on G_- and we therefore write

$$\tilde{R} = \mathcal{R}(G_-). \quad (19)$$

By definition, the mapping \mathcal{R} must satisfy²⁰

$$\mathcal{R}(G_-)_i = \mathbf{R}(G_-, \mathcal{R}(G_-)) \text{ for all } i \in [0, 1]. \quad (20)$$

We assume that $\mathcal{R}(G_-)$ is the unique fixed point of Eq. (20), which is indeed true in the linear-quadratic case. Eqs. (16) and (19) together form the law of motion of the aggregate states G and \tilde{R} corresponding to the given strategy \mathbf{g} . One can combine these two equations to obtain

$$G = \mathcal{H}(G_-) \quad (21)$$

where

$$\mathcal{H}(G_-) = \mathcal{G}(G_-, \mathcal{R}(G_-)). \quad (22)$$

This shows that the dynamics of the greenhouse gas stock can be separated from the accumulation of technology.

Now that we have determined the reaction of the countries to a fixed strategy \mathbf{g} and have described the induced dynamics of the aggregate state variables, we can evaluate the strategy \mathbf{g} according to the SEA's welfare measure. Let us denote by $V(G_-, \tilde{R}_-)$ the value of the strategy \mathbf{g} for a self of the authority that takes over control in a situation when the aggregate states are given by G_- and \tilde{R}_- . The value function must satisfy the recursive equation

$$\begin{aligned} V(G_-, \tilde{R}_-) = & \int_0^1 \left\{ B(\mathcal{R}(G_-)_i + \mathbf{g}(G_-, \mathcal{R}(G_-), \mathcal{R}(G_-)_i)) - C(\mathbf{g}(G_-, \mathcal{R}(G_-), \mathcal{R}(G_-)_i)) \right. \\ & \left. - D(\mathcal{H}(G_-)) - k[\mathcal{R}(G_-)_i - \rho R_{i,-}] \right\} di + \beta V(\mathcal{H}(G_-), \mathcal{R}(G_-)). \end{aligned}$$

It remains to determine the equilibrium policy function \mathbf{g} . To this end, we have to find out how off-equilibrium choices of the emission levels $\{g_i \mid i \in [0, 1]\}$ affect the countries' investments and future technology levels under the assumption that all future emission levels are chosen according to the equilibrium policy function \mathbf{g} . Consider a self that takes over control when the aggregate states are G_- and \tilde{R}_- . This self rationally anticipates that all its successors play the equilibrium strategy \mathbf{g} . Since the countries make their investment decisions before the SEA sets the emission levels, the self in charge can react

¹⁹ Whether the problem in (18) has a solution and, if so, whether this solution is unique depends among other things on the properties of the strategy \mathbf{g} . In the linear-quadratic example from Section 3.3 the strategy \mathbf{g} is a linear function and concavity of B together with strict convexity of C implies that there exists a unique solution.

²⁰ The mapping \mathbf{R} maps pairs (G_-, \tilde{R}) to the set of real numbers. The mapping \mathcal{R} , on the other hand, maps real numbers G_- to the set of distributions of technology levels over countries. We have chosen a notation that reflects these differences. The notation $\mathcal{R}(G_-)_i$ in the following Eq. (20) indicates the country- i value of the distribution $\mathcal{R}(G_-)$.

to off-equilibrium choices of the countries' technology levels. Thus the self under consideration needs to find the optimal emission levels $\{g_i | i \in [0, 1]\}$ for given aggregate states G_- and \tilde{R} . The individual countries, on the other hand, move before the SEA and, hence, their only rational expectation about the authority's behavior is the SEA's equilibrium strategy. Formally, even under off-equilibrium behavior of the authority, every country chooses its technology level such that (18)–(20) hold. To summarize, the self under consideration chooses the emission levels $\{g_i | i \in [0, 1]\}$ so as to maximize

$$\int_0^1 [B(R_i + g_i) - C(g_i) - D(G) - k(R_i - \rho R_{i,-})] di + \beta V(G, \tilde{R})$$

subject to (2). Dropping terms that do not depend on the emission levels and using (2) to eliminate G , it follows that the mapping $i \mapsto g_i = \mathbf{g}(G_-, \tilde{R}, R_i)$ maximizes the functional

$$\int_0^1 \left[B(R_i + g_i) - C(g_i) - D\left(\int_0^1 g_j dj + \gamma G_-\right) \right] di + \beta V\left(\int_0^1 g_j dj + \gamma G_-, \tilde{R}\right). \quad (23)$$

We are finally ready to define an equilibrium under discretion.

Definition 4. An equilibrium under discretion consists of functions $(\mathbf{g}, \mathbf{R}, \mathcal{G}, \mathcal{H}, \mathcal{R}, V)$ such that conditions (17), (18), (20), and (22)–(23) hold and such that, for all G_- , all \tilde{R} , and all R_i , the mapping $i \mapsto g_i = \mathbf{g}(G_-, \tilde{R}, R_i)$ maximizes (23).

We start the analysis of discretionary equilibria with the following important observation.

Theorem 1. The first-best solution cannot be implemented as an equilibrium under discretion.

To understand the intuition for Theorem 1 we make the following observations. First we see from Eq. (15) in Lemma 3 that for a given emission level g_i country i would indeed choose the corresponding efficient technology level. The second observation is that in an equilibrium under discretion, the emission level imposed by the SEA on any given country i must be responsive to the technology level chosen by that country, i.e., the strategy $g_i = \mathbf{g}(G_-, \tilde{R}, R_i)$ cannot be constant with respect to its last argument R_i . Consequently, the countries will no longer respond according to (15) because they will rationally anticipate that they can influence the SEA's emission policy. This changes the optimal response of country i ; see Eq. (47) in the appendix.

The separability and linearity assumptions made in Section 2 imply a number of useful properties of an equilibrium under discretion which we state in the following lemma. To this end, we denote by \bar{R}_- the average technology level of the distribution \tilde{R}_- , that is, $\bar{R}_- = \int_0^1 R_{i,-} di$.

Lemma 5. An equilibrium under discretion satisfies the following properties:

(a) The authority's equilibrium value function V is of the form

$$V(G_-, \tilde{R}_-) = V^*(G_-) + k\rho\tilde{R}_-.$$

(b) The equilibrium strategy \mathbf{g} is of the form

$$\mathbf{g}(G_-, \tilde{R}, R_i) = \mathbf{g}^*(G_-, R_i).$$

(c) The function \mathbf{R} is of the form

$$\mathbf{R}(G_-, \tilde{R}) = \mathbf{R}^*(G_-)$$

and $\mathcal{R}(G_-)_i = \mathbf{R}^*(G_-)$ holds for all $i \in [0, 1]$.

(d) The functions V^* , \mathbf{g}^* , and \mathbf{R}^* from parts (a)–(c) satisfy

$$V^*(G_-) = B(\mathbf{R}^*(G_-) + \mathbf{g}^*(G_-, \mathbf{R}^*(G_-))) - C(\mathbf{g}^*(G_-, \mathbf{R}^*(G_-))) - D(\mathcal{H}(G_-)) - (1 - \beta\rho)k\mathbf{R}^*(G_-) + \beta V^*(\mathcal{H}(G_-)) \quad (24)$$

and

$$B'(R_i + \mathbf{g}^*(G_-, R_i)) - C'(\mathbf{g}^*(G_-, R_i)) = D'(\mathcal{H}(G_-)) - \beta(V^*)'(\mathcal{H}(G_-)) \quad (25)$$

for all G_- and all R_i .

A further interesting property of the equilibrium is that the sensitivity of the authority's emission strategy \mathbf{g}^* with respect to the technology level R_i does not depend on the costs associated with global warming (i.e., the function D). This observation, which will become relevant in Section 5 below, is formalized in the following lemma.

Lemma 6. Suppose that the strategy \mathbf{g}^* is differentiable. Then it holds that $\partial \mathbf{g}^*(G_-, R_i) / (\partial R_i) \in (-1, 0)$ and that this partial derivative is independent of the form of the function D .

The intuition behind this result is the following. Since countries are infinitesimally small, a single country's emission and investment level only negligibly affects the aggregate stocks of greenhouse gases and clean technologies, respectively. Accordingly, they only negligibly affect the cost of global warming and the continuation value function of the authority, as established in Lemma 5. When deciding about changing the emission quota for a particular country in response to a change

in this country's technology level, the authority therefore only trades off the effect on the consumption benefit and the local air pollution cost for this country. As a result, the sensitivity of \mathbf{g}^* with respect to the technology level R_i depends only on the benefit function B and the local cost function C , but not on the global cost function D .

4.3. The linear-quadratic case

In this subsection we determine an equilibrium under discretion in the linear-quadratic environment introduced in Section 3.3.

Theorem 2. Consider the linear-quadratic specification from Section 3.3. There exists an equilibrium under discretion in which the policy functions take the forms

$$\mathbf{g}^*(G_-, R_i) = h_1 + h_G G_- + h_R R_i \quad (26)$$

and

$$\mathbf{R}^*(G_-) = R^{\text{BAU}}. \quad (27)$$

The coefficients h_1 , h_G , and h_R are given by

$$h_1 = \frac{(1+c)(z + \beta v_G) + (d - \beta v_{GG})R^{\text{BAU}}}{(1+c)(1+c+d - \beta v_{GG})}, \quad (28)$$

$$h_G = -\frac{(d - \beta v_{GG})\gamma}{1+c+d - \beta v_{GG}}, \quad (29)$$

$$h_R = -\frac{1}{1+c}, \quad (30)$$

where

$$v_{GG} = \frac{1+c+d - \beta(1+c)\gamma^2 - \sqrt{[1+c+d - \beta(1+c)\gamma^2]^2 + 4(1+c)\beta d\gamma^2}}{2\beta}, \quad (31)$$

$$v_G = \frac{\gamma(d - \beta v_{GG})R^{\text{BAU}}}{(1+c)(1 - \beta\gamma) + d - \beta v_{GG}}. \quad (32)$$

In this equilibrium, the stock of greenhouse gases converges monotonically to

$$G^{\text{DIS}} = \frac{(1+c)(1 - \beta\gamma)(1 - \beta\rho)k}{c[d + (1 - \gamma)(1 - \beta\gamma)(1+c)]}. \quad (33)$$

The equilibrium described in the above theorem has a number of interesting properties that we would like to point out.

First, in line with Lemma 6, the equilibrium strategy \mathbf{g}^* of the authority has a reaction coefficient that only depends on the local cost parameter, $h_R = -1/(1+c)$. The higher c , the less the authority adjusts a country's emission quota in response to the country's technology level. In particular, a higher c implies that the authority is less willing to grant higher emission quotas to countries that invest less in clean technologies.

Second, when the authority uses the equilibrium strategy \mathbf{g}^* given in (26), all countries choose the technology level R^{BAU} in period 0 and maintain that stock forever. To see this, recall that the optimal investment policy solves the maximization problem (18). In our linear-quadratic example, the first-order condition for the optimal choice of R_i thus boils down to

$$-[R_i + \mathbf{g}^*(G_-, R_i) - z] + \{ -[R_i + \mathbf{g}^*(G_-, R_i) - z] - c\mathbf{g}^*(G_-, R_i) \} h_R = (1 - \beta\rho)k.$$

The optimal investment choice equates the marginal benefit of investing into clean technologies to the marginal investment cost. The marginal benefit consists of two parts: the direct marginal benefit due to increased clean energy consumption, and the indirect marginal net benefit due to the authority's adjustment of the emission quota in response to R_i , which affects dirty energy consumption and thus local air pollution. When the authority's reaction coefficient is $h_R = -1/(1+c)$, as is stipulated by the authority's equilibrium policy function, the direct and indirect effects of the emission quota on the marginal benefit exactly cancel out. The marginal benefit is then independent of $\mathbf{g}^*(G_-, R_i)$ and, accordingly, independent of the aggregate state variables. Each country therefore chooses the same technology level in each time period independent of the stock of greenhouse gases, and this technology level is the same as in the BAU-equilibrium.

Third, given that in equilibrium the countries' investments are not responsive to any changes in the stock of greenhouse gases, the SEA can improve welfare only by curtailing emission rates relative to business as usual. As shown in the theorem, this results in the long-run stock of greenhouse gases to be equal to G^{DIS} stated in (33). It is easy to see that

$$G^{\text{FB}} < G^{\text{DIS}} < G^{\text{BAU}}$$

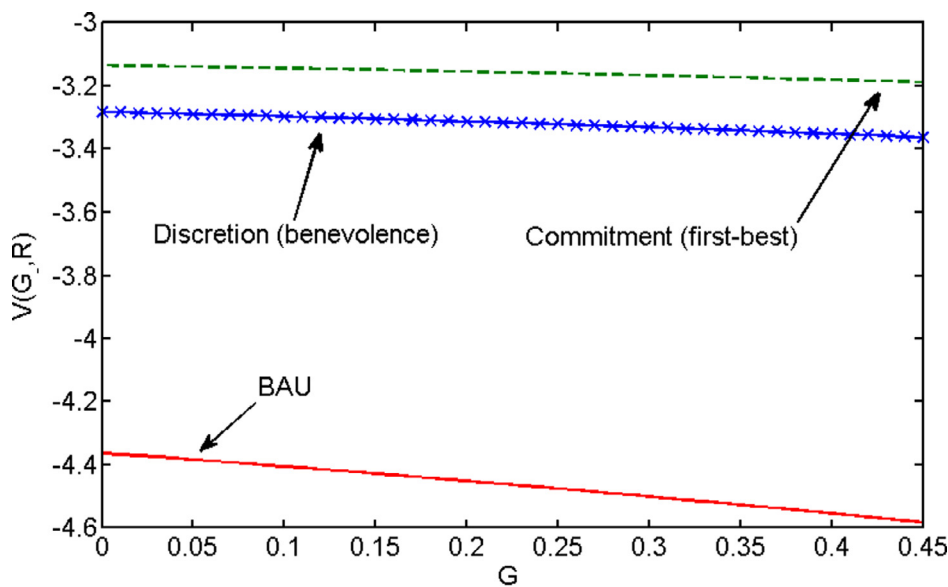


Fig. 1. Welfare comparisons.

holds whenever d is strictly positive.²¹ The following example provides a numerical illustration:²²

Example 1. Assume the functional forms stated in (14) with the following parameters: $k = 1$, $z = 1$, $\beta = 0.96$, $\rho = 0.9$, $\gamma = 0.9$, $c = 1$, and $d = 0.1$. Under these assumptions, the steady state in the BAU-equilibrium is given by $G^{\text{BAU}} = 1.36$ and $R^{\text{BAU}} = 0.728$, whereas the steady state in the first-best solution is $G^{\text{FB}} = 0.1628$ and $R^{\text{FB}} = 0.8477$. The steady state in a discretionary equilibrium with a benevolent SEA that has commitment power coincides with the first-best steady state. By contrast, the steady state in the equilibrium where emission decisions are delegated to a discretionary and benevolent SEA is given by $G^{\text{DIS}} = 0.2908$ and $R^{\text{DIS}} = 0.7280$.

Fig. 1 expands on Example 1 by plotting the welfare functions under business as usual, commitment, and discretion over the domain $G_- \in [0, 0.45]$.²³ As can be seen, delegation to a benevolent SEA noticeably improves welfare relative to the BAU equilibrium even when the authority lacks commitment.

5. Delegation to a non-benevolent authority

The delegation of emission control to an SEA can improve aggregate welfare relative to business as usual because the authority can internalize the external spillover effects of greenhouse gas emissions. However, as we have seen in Theorem 1, the authority cannot accelerate the conversion from fossil fuel based-energy production to emission free technologies if it lacks commitment power. As will be shown in the present section, the dynamic inconsistency problem at the root of this result can be mitigated by endowing the SEA with an objective function that differs from aggregate welfare (4). The idea of optimal delegation is borrowed from the literature on monetary policy, where dynamic inconsistency problems can be extenuated by augmenting the central bank's mandate by strong inflation aversion, as in Rogoff (1985), or by a desire for policy inertia, as in Woodford (2003).

5.1. Equilibrium definition

Let us for example assume that the objective function of the SEA is of the form

$$\sum_{t=0}^{+\infty} \beta^t \int_0^1 \hat{U}(G_t, R_{i,t}, g_{i,t}, r_{i,t}) di,$$

where

$$\hat{U}(G_t, R_{i,t}, g_{i,t}, r_{i,t}) = B(R_{i,t} + g_{i,t}) - kr_{i,t} - \hat{C}(g_{i,t}) - \hat{D}(G_t)$$

²¹ If d would be equal to 0, the steady state values G^{BAU} , G^{FB} , and G^{DIS} would coincide.

²² Our parameter choices are meant to be suggestive; as our model is very stylized, it is hard to assign the most realistic parameters.

²³ This choice of the domain is motivated by the property that the steady states under commitment and under discretion lie well within this interval.

and where \hat{C} and \hat{D} are strictly convex cost functions that may be different from the functions C and D describing the preferences of the countries. Of course, one could consider much more general mandates for the authority but the proposed form will be sufficient to make our point. We are particularly interested in the question of whether the SEA should attach more weight to the cost of local emissions $g_{i,t}$ or to the cost of the global stock of greenhouse gases G_t .

An equilibrium under discretion in this situation can be defined along the same lines as in the previous section. The only difference to the previous arguments is that one needs to take into account that the SEA evaluates its policy using the functions \hat{C} and \hat{D} . Analogously to [Section 4.2](#) we can therefore obtain the following equilibrium conditions.²⁴

Definition 5. An equilibrium under discretion consists of functions $(\mathbf{g}^*, \mathbf{R}^*, \mathcal{G}, \mathcal{H}, \mathcal{R}, \hat{V}^*)$ such that the following conditions hold:

$$\mathcal{G}(G_-, \bar{R}) = \int_0^1 \mathbf{g}^*(G_-, R_i) di + \gamma G_-, \quad (34)$$

$$\mathbf{R}^*(G_-) = \operatorname{argmax}_x \{B(x + \mathbf{g}^*(G_-, x)) - C(\mathbf{g}^*(G_-, x)) - (1 - \beta\rho)kx \mid x \in \mathbb{R}\}, \quad (35)$$

$$\mathcal{R}(G_-)_i = \mathbf{R}^*(G_-) \text{ for all } i, \quad (36)$$

$$\mathcal{H}(G_-) = \mathcal{G}(G_-, \mathcal{R}(G_-)), \quad (37)$$

$$\begin{aligned} \hat{V}^*(G_-) &= B(\mathbf{R}^*(G_-) + \mathbf{g}^*(G_-, \mathbf{R}^*(G_-))) - \hat{C}(\mathbf{g}^*(G_-, \mathbf{R}^*(G_-))) \\ &\quad - \hat{D}(\mathcal{H}(G_-)) - (1 - \beta\rho)k\mathbf{R}^*(G_-) + \beta\hat{V}^*(\mathcal{H}(G_-)), \end{aligned} \quad (38)$$

$$B'(R_i + \mathbf{g}^*(G_-, R_i)) - \hat{C}'(\mathbf{g}^*(G_-, R_i)) = \hat{D}'(\mathcal{H}(G_-)) - \beta(\hat{V}^*)'(\mathcal{H}(G_-)). \quad (39)$$

Note that the function \hat{V} defined by $\hat{V}(G_-, \bar{R}_-) = \hat{V}^*(G_-) + k\rho\bar{R}_-$ is the equilibrium value function of the authority. Aggregate welfare according to [\(4\)](#) is given by $V(G_-, \bar{R}_-) = V^*(G_-) + k\rho\bar{R}_-$, where V^* solves the recursive equation

$$\begin{aligned} V^*(G_-) &= B(\mathbf{R}^*(G_-) + \mathbf{g}^*(G_-, \mathbf{R}^*(G_-))) - C(\mathbf{g}^*(G_-, \mathbf{R}^*(G_-))) \\ &\quad - D(\mathcal{H}(G_-)) - (1 - \beta\rho)k\mathbf{R}^*(G_-) + \beta V^*(\mathcal{H}(G_-)). \end{aligned} \quad (40)$$

We would like to find out how the SEA's cost functions \hat{C} and \hat{D} must be chosen so that aggregate welfare is improved relative to the equilibrium under a benevolent SEA.

5.2. The linear-quadratic case

We consider again the same linear-quadratic specifications as introduced in [Section 3.3](#) and assume furthermore that the authority's costs functions take the forms

$$\hat{C}(g) = \frac{\hat{c}g^2}{2}, \quad \hat{D}(G) = \frac{\hat{d}G^2}{2},$$

where \hat{c} and \hat{d} are positive parameters. On first thought, one may think that the value of \hat{c} is of subordinate importance but that the authority should be given a high value of \hat{d} . After all, the external effect operates via the global stock of greenhouse gases, and it is tempting to suggest that this stock must be highly penalized. The following analysis, however, proves that this conjecture is wrong and shows that the parameter \hat{c} of the cost of local emissions plays the key role.

5.2.1. Variations in the authority's assessment of global warming costs, \hat{d}

We start by considering the scenario where the authority shares the countries' assessment of local air pollution costs, $\hat{c} = c$, but its assessment of global warming costs may differ from the countries' assessment. Formally, we allow \hat{d} to be an arbitrary positive number, and examine whether aggregate world welfare can be improved relative to the benevolent case by assigning the authority a value $\hat{d} \neq d$. The following lemma characterizes the discretionary equilibrium.

Lemma 7. Assume that $\hat{c} = c$ and $\hat{d} > 0$. There exists an equilibrium under discretion in which the strategy \mathbf{g}^* has the form

$$\mathbf{g}^*(G_-, R_i) = \hat{h}_1 + \hat{h}_G G_- + \hat{h}_R R_i, \quad (41)$$

²⁴ Note, in particular, that a result analogous to [Lemma 5](#) holds also in the present situation.

and in which (27) holds. The coefficients \hat{h}_1 , \hat{h}_G , and \hat{h}_R are given by the same formulas as the corresponding coefficients h_1 , h_G , and h_R in Theorem 2 except that the parameter d is replaced by \hat{d} wherever it appears. The stock of greenhouse gases converges monotonically to

$$\hat{G}^{\text{DIS}} = \frac{(1 - \beta\gamma)(1 - \beta\rho)(1 + c)k}{c[\hat{d} + (1 - \gamma)(1 - \beta\gamma)(1 + c)]}. \quad (42)$$

An important implication of Lemma 7 is that the authority's reaction coefficient \hat{h}_R is the same as if the authority was benevolent. The incentives of countries to invest in clean technologies are thus not affected by changes in \hat{d} , which further implies that the authority's dynamic inconsistency problem cannot be mitigated by changes in \hat{d} . As the poor welfare in the discretionary equilibrium is not caused by the externality per se, but by the dynamic inconsistency, this suggests that welfare cannot be improved by variations of \hat{d} alone. This reasoning leads to the following theorem.²⁵

Theorem 3. Let $\hat{d} > 0$ be arbitrarily given. Aggregate welfare in the discretionary equilibrium with a non-benevolent SEA of type (c, \hat{d}) is smaller than or equal to aggregate welfare in the discretionary equilibrium with a benevolent SEA of type (c, d) (which is described in Theorem 2).

What, then, is the role of the parameter \hat{d} in the present example? It is obvious from Eq. (42) that by varying \hat{d} between 0 and $+\infty$ one can generate any steady state greenhouse gas stock between 0 and

$$G^{\text{BAU}} = \frac{(1 - \beta\rho)k}{(1 - \gamma)c}.$$

For example, by choosing $\hat{d} = (1 + c)d/c > d$ one obtains the first-best steady state G^{FB} . One could also try to find that value of \hat{d} which maximizes per-period welfare in steady state. To this end, recall from Lemma 7 that independently of the value of \hat{d} it holds in the discretionary equilibrium for all $i \in [0, 1]$ and all $t \geq 0$ that $R_{i,t} = R^{\text{BAU}}$. Furthermore, in steady state we must have $g_{i,t} = (1 - \gamma)G$. When one maximizes per-period welfare across those steady states for which $R_{i,t} = R^{\text{BAU}}$ holds, one must therefore choose G in such a way that

$$B(R^{\text{BAU}} + (1 - \gamma)G) - C((1 - \gamma)G) - D(G)$$

is maximized. One might call this a restricted Golden Rule, where the word “restricted” indicates that $R_{i,t} = R^{\text{BAU}}$ is imposed. Solving the first-order condition for this problem yields

$$G = \frac{(1 - \gamma)(z - R^{\text{BAU}})}{d + (1 - \gamma)^2(1 + c)},$$

which can be achieved by setting $\hat{d} = (1 - \beta\gamma)d/(1 - \gamma) > d$. To summarize, by fixing the value of \hat{c} at c and varying the cost parameter \hat{d} only, one can steer the stock of greenhouse gases towards its first-best steady state or other desirable values (like the restricted Golden Rule stock) but one cannot achieve higher welfare than the benevolent SEA would implement.

5.2.2. Variations in the authority's assessment of local air pollution costs, \hat{c}

We next consider the scenario where the authority's assessment of local air pollution costs may differ from the countries' assessment, $\hat{c} > 0$, while maintaining the assumption that $\hat{d} = d$. In the appendix, we show that there exists a discretionary equilibrium where the authority's strategy is again of the form

$$\mathbf{g}^*(G_-, R_i) = \hat{h}_1 + \hat{h}_G G_- + \hat{h}_R R_i,$$

with a reaction coefficient²⁶

$$\hat{h}_R = -\frac{1}{1 + \hat{c}}.$$

The countries investment strategy, in turn, is given by

$$\mathbf{R}^*(G_-) = \frac{(1 + \hat{c})[\hat{h}_1(c - \hat{c}) - (1 - \beta\rho)k(1 + \hat{c}) + \hat{c}z]}{c + \hat{c}^2} + \frac{(1 + \hat{c})\hat{h}_G(c - \hat{c})}{c + \hat{c}^2} G_-.$$

Note that the reaction coefficient \hat{h}_R is directly affected by changes in \hat{c} . In particular, assigning a higher \hat{c} to the authority results in a less responsive emission strategy. This, in turn, increases the countries' incentive to invest in clean technologies, because higher investment leads to a less severe reduction in the emission quota. Increasing \hat{c} thus mitigates the authority's dynamic inconsistency problem, and provides incentives for countries to invest more than in the BAU-equilibrium. The following example provides a numerical illustration.

²⁵ It will be convenient to refer to the pair (\hat{c}, \hat{d}) as the *type* of the SEA. The benevolent authority, for example, is of type (c, d) .

²⁶ We relegate the relevant formulas for the parameters \hat{h}_1 and \hat{h}_G to the appendix, as the algebra is relatively cumbersome.

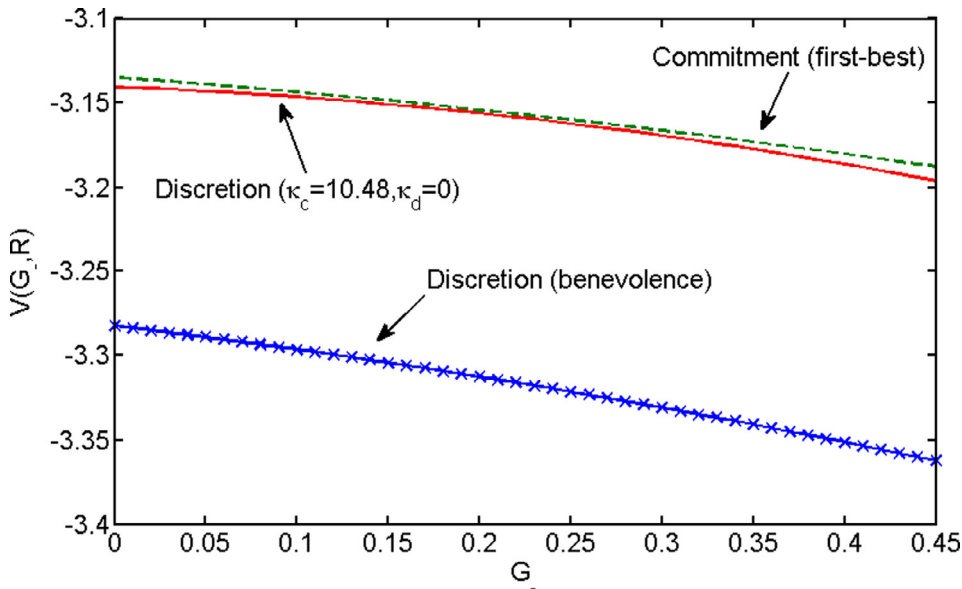


Fig. 2. Welfare comparison: commitment versus discretion. Note: $k_c = \hat{c}/c$ and $k_d = \hat{d}/d$.

Example 2. Assume the same parameters as in Example 1 and $\hat{d} = d$. Under this assumption, the optimal weight on local air pollution assigned to the discretionary SEA, i.e., the parameter \hat{c} that maximizes aggregate welfare over the domain $G_- \in [0, 0.45]$ is given by $\hat{c} = 7.22$. The steady state in a discretionary equilibrium with an authority of type $(\hat{c}, \hat{d}) = (7.22, 0.1)$ is given by $\hat{G}^{DIS} = 0.1084$ and $\hat{R}^{DIS} = 0.8358$.

Note that the long-run stock of greenhouse gases is even lower than in the first-best allocation. This is because a higher weight on local air pollution costs not only mitigates the authority's dynamic inconsistency problem, but also distorts its trade-off between dirty and clean energy consumption towards clean energy. This suggests that when $\hat{c} > c$, further welfare improvements may be feasible by lowering the authority's assessment of global warming costs, \hat{d} , to mitigate this distortion. Indeed, somewhat surprisingly, optimal delegation under discretion in our environment requires the authority to place a large weight on local air pollution costs and zero weight on global warming.

Example 3. Assume the same parameters as in Example 1. The optimal type of the discretionary SEA, i.e., the pair of parameters (\hat{c}, \hat{d}) that maximizes aggregate welfare over the domain $G_- \in [0, 0.45]$ is given by $\hat{c} = 10.48$ and $\hat{d} = 0$. The steady state in a discretionary equilibrium with an authority of type $(\hat{c}, \hat{d}) = (10.48, 0)$ is given by $\hat{G}^{DIS} = 0.1409$ and $\hat{R}^{DIS} = 0.8382$.

Comparing Examples 2 and 3, we see that reducing the weight on global warming costs all the way to zero makes it optimal to assign an even higher weight on local air pollution. As has already been indicated, this is the case because operating under such a mandate alleviates the distortion between clean and dirty energy consumption. The latter is also reflected in the long-run stocks of greenhouse gases and clean technologies, which are now significantly closer to their first-best values. Note finally that delegation of emission policies to an optimally designed discretionary SEA eliminates almost the entire welfare loss resulting from lack of commitment, as is visualized in Fig. 2.

5.2.3. Optimal mandates and state-dependency

So far we have defined *optimal* mandates as those parameter combinations (\hat{c}, \hat{d}) that maximize the aggregate world welfare over a relevant domain in the state space. However, as we show next, the optimal mandate is in fact dependent on the state of the economy, (G_-, R_-) , and hence it is not time-invariant as the stock of greenhouse gases evolves over time. Rather, for any initial state, the optimal type of the SEA is given by

$$\operatorname{argmin}_{\hat{c}, \hat{d}} |V^{FB}(G_-, R_-) - V^{DIS}(G_-, R_- | \hat{c}, \hat{d})|. \quad (43)$$

Fig. 3 displays the optimal SEA type as a function of G_- , while holding R_- constant, under the parametric assumptions of Example 1. When G_- is not too large, the optimal choice is to set $\hat{c} > c$ and $\hat{d} < d$, in line with our discussion of Example 3. If, however, the initial stock of greenhouse gases is very large relative to the steady state, then it is optimal to also put a larger weight on the costs of global warming, $\hat{d} > d$. In such a situation, the optimal authority may be labelled *environmentalist*, i.e. it places a higher weight on both local and global costs of pollution relative to society. This allows for a fast reduction of greenhouse gases in the atmosphere.

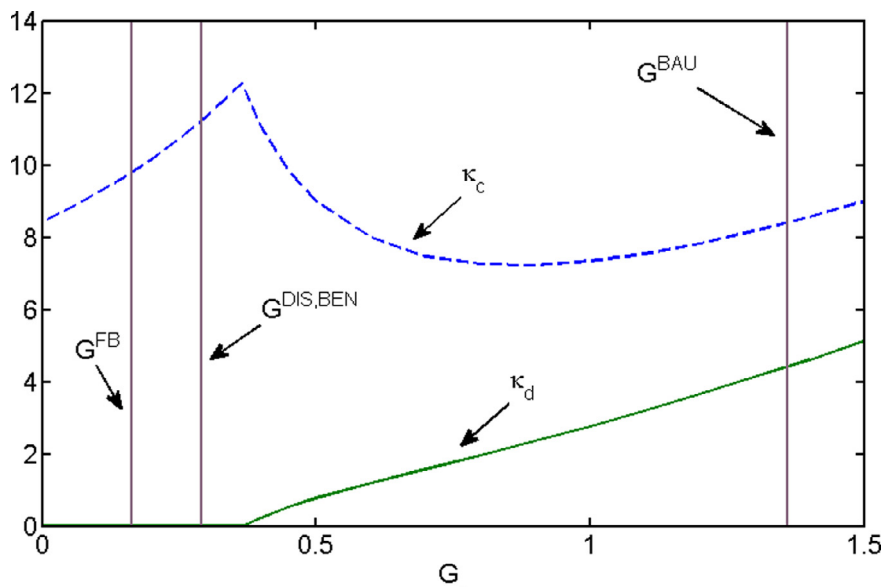


Fig. 3. Optimal mandates. *Note:* The figure visualizes optimal mandates relative to the preference parameters of society, $k_c = \hat{c}/c$ and $k_d = \hat{d}/d$. Note that since $c = 1$ and $d = 0.1$, $\hat{c} = k_c$ and $\hat{d} = k_d/10$.

The above finding points to a further complication of the optimal specification of an SEA's mandate. If the countries establish at time 0 (when the stock of greenhouse gases is equal to G_{-1}) an SEA with optimal type (\hat{c}, \hat{d}) , they will face an incentive to revise this mandate when the greenhouse gas concentration changes in the course of time. One can think of two possible ways to approach this situation. One possibility is to allow for state-dependent mandates, that is, for SEA types $(\hat{c}(G_-), \hat{d}(G_-))$. A SEA which is endowed with such a mandate would of course anticipate that it has to use different evaluations of the respective local and global costs if the greenhouse gas concentration changes. The strategies derived above would therefore no longer be an equilibrium. Another possible approach would be to maintain the assumption of state-independent mandates (\hat{c}, \hat{d}) but to take the incentives of the countries for a revision of the mandate into account. In other words, one would have to address yet another dynamic inconsistency problem, this time one that pertains to the decision over the mandate of the SEA made by the countries. We elaborate on this issue in the following section.

6. Discretionary delegation

We now assume that the countries decide on the SEA's mandate in a discretionary fashion, a situation we refer to as discretionary delegation. At the start of every time period, the countries jointly choose the mandate (\hat{c}, \hat{d}) for the current authority. Subsequently they make their individual investment decisions and, finally, the SEA chooses the emission levels assigned to all countries. To simplify the exposition we restrict attention to the linear quadratic case.

In the environment under consideration, the mandate given to the authority will be a function of the stock of greenhouse gases, G_- . We refer to this function as a *mandate rule* and use the notation $(\hat{c}, \hat{d}) = M(G_-)$.²⁷ To determine the equilibrium mandate rule we proceed in three steps. First, we define equilibrium for a given arbitrary mandate rule M , which allows us to determine the welfare levels of the countries and the authority conditional on the mandate rule M being in place forever. Second, we determine the optimal current mandate $m = (\hat{c}, \hat{d})$ under the assumption that future mandates are determined by the rule M . Since the optimal current mandate depends on the current stock of greenhouse gases G_- , this step defines an optimal current mandate rule for any given future mandate rule, i.e., $m = \hat{M}(G_-; M)$. Finally, we define the equilibrium mandate rule M^* as the fixed-point satisfying $\hat{M}(G_-; M^*) = M^*(G_-)$.²⁸

The equilibrium under an arbitrary mandate rule M can be characterized in analogy to Definition 5 so that we do not present further details here. Let us denote the value functions of the SEA and the countries in this equilibrium by

$$\begin{aligned}\hat{V}(G_-, \tilde{R}_-; M) &= \hat{V}^*(G_-; M) + k\rho\tilde{R}_-, \\ V(G_-, \tilde{R}_-; M) &= V^*(G_-; M) + k\rho\tilde{R}_-, \end{aligned}$$

where the notation we use makes explicit that the value functions depend on the mandate rule in place.

²⁷ By taking the limit of a finite horizon economy when the time horizon approaches infinity, it can be shown that in equilibrium the mandate rule is independent of \tilde{R}_- .

²⁸ Note that the equilibrium mandate rule derived in this way is time-consistent since the countries do not have an incentive to change the rule today if they perceive that this rule is in place from tomorrow onwards.

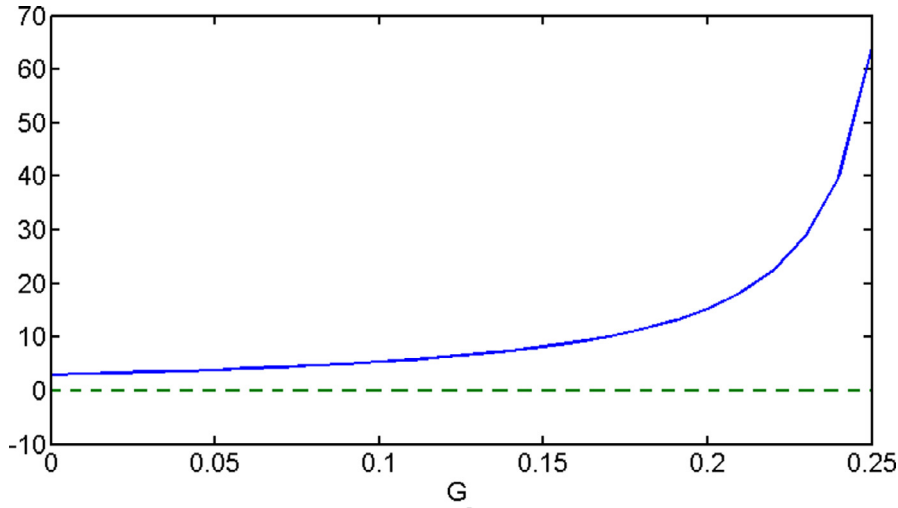


Fig. 4. Optimal mandate rule under discretionary delegation. *Note:* The blue solid line gives the optimal mandate rule for \hat{c} and the green dashed line for \hat{d} . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Let us now consider the optimal emission policies by a self of the SEA that is endowed with the mandate $m = (\hat{c}, \hat{d})$ and perceives future mandates to be determined by the rule M . This self chooses the emission levels $\{g_i | i \in [0, 1]\}$ so as to maximize

$$\int_0^1 \left[-\frac{1}{2}(R_i + g_i - z)^2 - \frac{\hat{c}}{2}g_i^2 - \frac{\hat{d}}{2}G^2 \right] di + \beta \hat{V}^*(G; M)$$

subject to (2). Let the solution to this maximization problem be denoted by $\mathbf{g}_i(G_-, R_i; m, M)$, $i \in [0, 1]$. Given the current mandate m and the policies $\mathbf{g}_i(G_-, R_i; m, M)$, the countries' optimal investment decisions can be derived as

$$\mathbf{R}(G_-; m, M) = \operatorname{argmax}_x \left\{ -\frac{1}{2}[x + \mathbf{g}_i(G_-, x; m, M) - z]^2 - \frac{c}{2}\mathbf{g}_i(G_-, x; m, M)^2 - (1 - \beta\rho)kx \mid x \in \mathbb{R} \right\}.$$

Note that, as before, all countries choose the same technology level, independently of their initial stock $R_{i,-}$. Hence all countries will also receive the same emission quota,

$$\mathbf{g}(G_-; m, M) = \mathbf{g}_i(G_-, \mathbf{R}(G_-; m, M); m, M).$$

Aggregate world welfare, conditional on the current mandate m and the future mandate rule M , can be expressed as

$$\begin{aligned} V^*(G_-; m, M) + k\rho\bar{R}_- &= -\frac{1}{2}[\mathbf{R}(G_-; m, M) + \mathbf{g}(G_-; m, M) - z]^2 - \frac{c}{2}\mathbf{g}(G_-; m, M)^2 - \frac{d}{2}[\mathbf{g}(G_-; m, M) + \gamma G_-]^2 \\ &\quad - k[\mathbf{R}(G_-; m, M) - \rho\bar{R}_-] + \beta V^*(\mathbf{g}(G_-; m, M) + \gamma G_-; M) + \beta k\rho\mathbf{R}(G_-; m, M). \end{aligned}$$

The optimal current mandate rule given the future mandate rule M can thus be computed as

$$\hat{M}(G_-; M) = \operatorname{argmax}_{(\hat{c}, \hat{d})} \left\{ V^*(G_-; (\hat{c}, \hat{d}), M) \mid (\hat{c}, \hat{d}) \in \mathbb{R}_+^2 \right\}.$$

Finally, the equilibrium mandate rule is determined as the fixed-point satisfying $M^*(G_-) = \hat{M}(G_-; M^*)$.

Example 4. Assume the same parameters as in Example 1. The steady state in the equilibrium with discretionary delegation is given by $G^{DD} = 0.1485$ and $R^{DD} = 0.8342$. In this steady state, the mandate given to the supranational environmental authority is $\hat{c} = 8.0784$, $\hat{d} = 0$.

Fig. 4 expands on Example 4 by illustrating the mandate rule in the neighborhood of the steady state. It shows that the optimal mandate rule for \hat{c} is strongly increasing in the stock of greenhouse gases G_- , whereas the optimal mandate rule for \hat{d} is flat at $\hat{d} = 0$.

Finally, Fig. 5 compares world welfare under discretionary delegation (blue dashed line) to the welfare levels in the first best (red line) and under optimal delegation to a non-benevolent authority with a time-invariant mandate (green line). Quantitatively, differences between the welfare levels under discretionary delegation and optimal time-invariant delegation are very small. Interestingly, the welfare functions intersect. This is possible because there are two mechanisms at work that

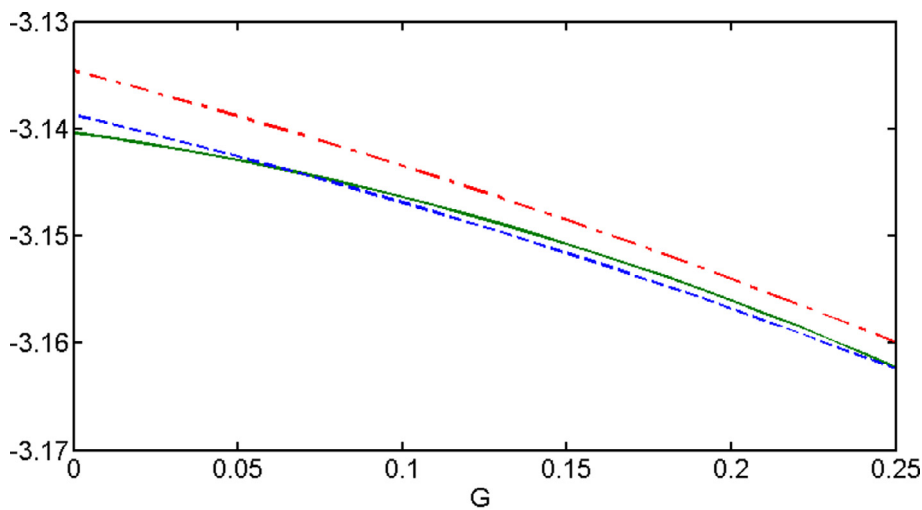


Fig. 5. Discretionary delegation: welfare. *Note:* Aggregate welfare under optimal delegation to a non-benevolent authority with a time-invariant mandate (green solid line), under discretionary delegation (blue dashed line), and in the first best (red dash-dotted line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

are responsible for the welfare differences. First, in our environment with discretionary delegation the mandate is allowed to depend on the state of the economy. This greater flexibility compared to the case of a constant, time-invariant mandate leads to welfare gains. Second, the countries cannot commit over time to an optimal mandate, which leads to welfare losses relative to the case where countries commit to a constant, time-invariant mandate. These two opposing effects determine the welfare differences observed in Fig. 5.

7. Concluding remarks

Dynamic inconsistency is ubiquitous in economic policy, and environmental policy is no exception. In the present paper we have pointed out that such a problem would also occur if a supranational environmental authority were established in order to control the emission of greenhouse gases in a world consisting of many individual countries. We have used a very simple and stylized, yet fully dynamic model to make our point. The main result is that the welfare loss that results from lack of commitment power can be kept very small by endowing the authority with a mandate to heavily penalize the local costs of emissions and to attach only little weight to the cost of global warming. However, assigning such a mandate to the authority is again dynamically inconsistent, as countries face a recurrent incentive to modify the authority's mandate over time. The foundation of an SEA with full decision power over emission quotas is hence subject to several obstacles, which may help explain why such an authority has not yet been implemented in practice.

We arrive at this conclusion even though our analysis abstracts from several important real-world difficulties. First, we have abstracted from any uncertainty regarding the true economic costs of air pollution and global warming. Second, we have not studied the question of why the countries would want to install the SEA and comply with its decisions in the first place; we simply assumed that the authority exists and that the countries have delegated some of their decision power to it. Third, we have assumed that all countries are infinitesimally small and symmetric regarding their preferences, and hence they have no conflict of interest when deciding about the authority's mandate. Finally, we have just considered a very restricted set of possible mandates for the SEA, while many other alternative mandates could be considered. Addressing all these issues is both interesting and important. Yet this goes beyond the scope of the present analysis and is left for future research.

Appendix

Proof of Lemma 1

Consider an arbitrary country $i \in [0, 1]$. Since this country takes $(G_t)_{t=0}^{+\infty}$ as given, one can drop the additively separated terms $-\beta^t D(G_t)$ from its objective function (3) and it follows from (1) that country i maximizes

$$\sum_{t=0}^{+\infty} \beta^t [B(R_{i,t} + g_{i,t}) - C(g_{i,t}) - k(R_{i,t} - \rho R_{i,t-1})] = k\rho R_{i,-1} + \sum_{t=0}^{+\infty} \beta^t [B(R_{i,t} + g_{i,t}) - C(g_{i,t}) - (1 - \beta\rho)kR_{i,t}]$$

with respect to $(R_{i,t}, g_{i,t})_{t=0}^{+\infty}$. Obviously, this is the case if and only if

$$B'(R_{i,t} + g_{i,t}) = C'(g_{i,t}) = (1 - \beta\rho)k \quad (44)$$

holds for all $i \in [0, 1]$ and all $t \geq 0$. Since the latter condition does neither explicitly depend on the country index i nor on time t , we obtain (5)–(6). Finally, by combining (2) and (5) one obtains (7).

Proof of Lemma 2

Using (1) we can rewrite (4) as

$$\begin{aligned} & \sum_{t=0}^{+\infty} \beta^t \int_0^1 [B(R_{i,t} + g_{i,t}) - C(g_{i,t}) - D(G_t) - k(R_{i,t} - \rho R_{i,t-1})] di \\ &= k\rho \int_0^1 R_{i,-1} di + \sum_{t=0}^{+\infty} \beta^t \int_0^1 [B(R_{i,t} + g_{i,t}) - C(g_{i,t}) - D(G_t) - (1 - \beta\rho)kR_{i,t}] di. \end{aligned}$$

This expression is maximized with respect to the variables $R_{i,t}$ for all $i \in [0, 1]$ and all $t \geq 0$ if and only if

$$B'(R_{i,t} + g_{i,t}) = (1 - \beta\rho)k, \quad (45)$$

which implies $B(R_{i,t} + g_{i,t}) - (1 - \beta\rho)kR_{i,t} = B(A) - (1 - \beta\rho)k(A - g_{i,t})$, where $A = (B')^{-1}((1 - \beta\rho)k)$. It follows therefore that the family of sequences $\{(g_{i,t})_{t=0}^{+\infty} | i \in [0, 1]\}$ has to be chosen so as to maximize

$$\sum_{t=0}^{+\infty} \beta^t \int_0^1 [(1 - \beta\rho)kg_{i,t} - C(g_{i,t}) - D(G_t)] di \quad (46)$$

subject to (2). Since the emission rates $g_{i,t}$ enter the constraint (2) only via the integral $\int_0^1 g_{i,t} di$ and since the cost function C is strictly convex, it follows that for every fixed $t \geq 0$ all countries' emission levels $g_{i,t}$ must be equal. Denoting the common value by g_t it follows from (45) that (9)–(11) hold. The Lagrangian function of the optimization problem is therefore

$$L = \sum_{t=0}^{+\infty} \beta^t \{ (1 - \beta\rho)kg_t - C(g_t) - D(G_t) + \lambda_t[g_t + \gamma G_{t-1} - G_t] \},$$

where λ_t denotes the Lagrange multiplier corresponding to constraint (2). Writing down the necessary and sufficient first-order conditions plus transversality condition and eliminating the Lagrange multipliers from these conditions yields (12) and (13).

Proof of Lemma 3

Given $(G_t)_{t=0}^{+\infty}$ and $\{(g_{j,t})_{t=0}^{+\infty} | j \in [0, 1]\}$, the maximization of (3) with respect to $(R_{i,t})_{t=0}^{+\infty}$ is equivalent to the maximization of

$$\sum_{t=0}^{+\infty} \beta^t [B(R_{i,t} + g_{i,t}) - k(R_{i,t} - \rho R_{i,t-1})] = k\rho R_{i,-1} + \sum_{t=0}^{+\infty} \beta^t [B(R_{i,t} + g_{i,t}) - (1 - \beta\rho)kR_{i,t}]$$

with respect to $(R_{i,t})_{t=0}^{+\infty}$. Obviously, the latter expression is maximized if and only if (15) is satisfied.

Proof of Lemma 4

Condition (15) coincides with condition (45). Using the same arguments as in the proof of Lemma 2 it can be seen that the authority's optimization problem is equivalent to the maximization of (46) subject to (2). Since this is the same problem that a social planner would solve, the proof of the lemma is complete.

Proof of Theorem 1

The first-order condition associated with problem (18) requires that in any equilibrium under discretion it holds for all $i \in [0, 1]$ that

$$0 = B'(R_i + \mathbf{g}(G_-, \tilde{R}, R_i)) - (1 - \beta\rho)k + [B'(R_i + \mathbf{g}(G_-, \tilde{R}, R_i)) - C'(\mathbf{g}(G_-, \tilde{R}, R_i))] \frac{\partial \mathbf{g}(G_-, \tilde{R}, R_i)}{\partial R_i}. \quad (47)$$

Now suppose that the first-best solution is an equilibrium under discretion. Combining the above condition with (11) one obtains

$$0 = [B'(R_i + \mathbf{g}(G_-, \tilde{R}, R_i)) - C'(\mathbf{g}(G_-, \tilde{R}, R_i))] \frac{\partial \mathbf{g}(G_-, \tilde{R}, R_i)}{\partial R_i}.$$

We claim that this cannot be the case. Suppose first that $\partial \mathbf{g}(G_-, \tilde{R}, R_i) / (\partial R_i) = 0$. In this case, the authority's equilibrium policy would assign the same emission level to all countries independent of their technology level R_i , that is, there would exist a value g such that the optimal solution of problem (23) satisfies $g_i = g$ for all $i \in [0, 1]$. Holding the total amount of emissions $\int_0^1 g_j dj$ constant at the optimal level g , a reallocation of emission rights from countries with larger technology levels to those with smaller ones would improve world welfare because B is concave. Formally, this can be seen from the first-order condition for the maximization of (23) subject to the constraint $\int_0^1 g_j dj = g$. As a matter of fact, this condition is

$$B'(R_i + g) - C'(g) + \lambda = 0,$$

where λ is the Lagrange multiplier of the constraint. Obviously, this equation cannot hold for all $i \in [0, 1]$ if there exist countries with different technology levels.²⁹

Now assume that $B'(R_i + \mathbf{g}(G_-, \tilde{R}, R_i)) - C'(\mathbf{g}(G_-, \tilde{R}, R_i)) = 0$ is satisfied along the first-best equilibrium path. Then it follows from Eqs. (11) and (12) that $D'(G_i) = 0$ holds along this path, which cannot be true as D is a strictly convex function. This completes the proof.

Proof of Lemma 5

The result in (a) follows immediately from Eq. (23). Using the result from part (a), the result in (b) follows easily from Eq. (23) and the result in (c) follows from (18) and (20). Due to the results in (a) and (b), Eq. (23) simplifies to (24). A necessary and sufficient first-order condition for the maximization of (23) with respect to $\{g_i | i \in [0, 1]\}$ is

$$B'(R_i + \mathbf{g}^*(G_-, R_i)) - C'(\mathbf{g}^*(G_-, R_i)) = D' \left(\int_0^1 \mathbf{g}^*(G_-, R_j) dj + \gamma G_- \right) - \beta (V^*)' \left(\int_0^1 \mathbf{g}^*(G_-, R_j) dj + \gamma G_- \right)$$

for all $i \in [0, 1]$, where we have utilized the results from parts (a) and (b). Applying part (b) once more and combining it with (17), (19), and (22) we obtain (25).

Proof of Lemma 6

Differentiating Eq. (25) with respect to R_i it follows that:

$$B''(R_i + \mathbf{g}^*(G_-, R_i)) [1 + \partial \mathbf{g}^*(G_-, R_i) / \partial R_i] = C''(\mathbf{g}^*(G_-, R_i)) \partial \mathbf{g}^*(G_-, R_i) / \partial R_i,$$

which yields

$$\frac{\partial \mathbf{g}^*(G_-, R_i)}{\partial R_i} = \frac{B''(R_i + \mathbf{g}^*(G_-, R_i))}{C''(\mathbf{g}^*(G_-, R_i)) - B''(R_i + \mathbf{g}^*(G_-, R_i))}.$$

Since B is concave and C is strictly convex, the above formula proves the first claim. Since the above formula does not involve the function D , the second claim holds as well.

Proof of Theorem 2

Due to the linear-quadratic structure of the problem, we guess that the equilibrium strategy \mathbf{g} is linear and that the equilibrium value function V is quadratic in G_- . According to Lemma 5 this means that (26) and

$$V^*(G_-) = v_1 + v_G G_- + \frac{v_{GG}}{2} (G_-)^2$$

must hold, where h_1 , h_G , h_R , v_1 , v_G , and v_{GG} are undetermined coefficients. Solving Eq. (25) for $\mathbf{g}^*(G_-, R_i)$ yields

$$\mathbf{g}^*(G_-, R_i) = \frac{z - R_i - d\mathcal{H}(G_-) + \beta[v_G + v_{GG} \mathcal{H}(G_-)]}{1 + c}, \quad (48)$$

which proves (30). Using (30) it is straightforward to solve (18) and we obtain

$$\mathbf{R}^*(G_-) = z - \frac{1+c}{c} (1 - \beta\rho)k = R^{\text{BAU}},$$

which proves (27). Condition (20) implies that the function \mathcal{R} is constant as well. Its constant value is the degenerate distribution with

$$\mathcal{R}(G_-)_i = R^{\text{BAU}} \quad (49)$$

²⁹ Note that we utilize the property of Markov-perfection. As we have shown, all countries choose identical technology levels along the equilibrium path. However, Markov-perfection requires that the authority's strategy is a best response also off the equilibrium path.

for all $i \in [0, 1]$. From (17) and (26) we obtain

$$\mathcal{G}(G_-, \tilde{R}) = h_1 + (h_G + \gamma)G_- - \frac{\tilde{R}}{1+c}$$

and from (22) and (49) it follows therefore that

$$\mathcal{H}(G_-) = h_1 + (h_G + \gamma)G_- - \frac{R^{\text{BAU}}}{1+c}. \quad (50)$$

Substituting this into (48) and equating the coefficients of the powers of G_- on both sides yields two linear equations for h_1 and h_G . These equations have a solution if and only if

$$v_{GG} \neq \frac{1+c+d}{\beta}. \quad (51)$$

If (51) holds, the unique solution is given by (28) and (29).

It remains to determine the coefficients v_1 , v_G , and v_{GG} . To this end, we evaluate (24). Using the above results, this equation can be written as

$$\begin{aligned} & v_1 + v_G G_- + \frac{v_{GG}}{2} (G_-)^2 \\ &= -\frac{1}{2} \left(h_1 + h_G G_- + \frac{cR^{\text{BAU}}}{1+c} \right)^2 - \frac{c}{2} \left(h_1 + h_G G_- - \frac{R^{\text{BAU}}}{1+c} \right)^2 - \frac{d}{2} \left[h_1 + (h_G + \gamma)G_- - \frac{R^{\text{BAU}}}{1+c} \right]^2 \\ & \quad - (1 - \beta\rho)kR^{\text{BAU}} + \beta \left\{ v_1 + v_G \left[h_1 + (h_G + \gamma)G_- - \frac{R^{\text{BAU}}}{1+c} \right] + \frac{v_{GG}}{2} \left[h_1 + (h_G + \gamma)G_- - \frac{R^{\text{BAU}}}{1+c} \right]^2 \right\}. \end{aligned} \quad (52)$$

Equating the coefficients of G_- on both sides of Eq. (52) and replacing h_1 and h_G by the values from (28) and (29) one obtains (32). Equating the coefficients of G_-^2 on both sides of (52) yields

$$v_{GG} = -\frac{(1+c)h_G^2 + d(h_G + \gamma)^2}{1 - \beta(h_G + \gamma)^2}.$$

Substituting for h_G from (29) and noting (51), it follows that

$$\beta v_{GG}^2 - [1+c+d - \beta(1+c)\gamma^2]v_{GG} - (1+c)d\gamma^2 = 0.$$

This quadratic equation has one negative and one positive root. Since the equilibrium value function must be concave, we consider only the negative root, which is given by (31). Since v_{GG} is negative, condition (51) is trivially satisfied.

Next, we compute the steady state values G^{DIS} , R^{DIS} , and g^{DIS} that correspond to the above equilibrium. We already know that $R^{\text{DIS}} = R^{\text{BAU}}$. Using the fact that \mathbf{R}^* is a constant function and that $\partial \mathbf{g}^*(G_-, R_i)/(\partial G_-) = h_G$ and $\partial \mathcal{H}(G_-)/(\partial G_-) = h_G + \gamma$, it follows from (24) by differentiating and evaluating at the steady state that

$$(V^*)'(G^{\text{DIS}}) = [B'(R^{\text{BAU}} + g^{\text{DIS}}) - C'(g^{\text{DIS}})]h_G - [D'(G^{\text{DIS}}) - \beta(V^*)'(G^{\text{DIS}})](h_G + \gamma).$$

Furthermore, evaluation of (25) at the steady state leads to

$$B'(R^{\text{BAU}} + g^{\text{DIS}}) - C'(g^{\text{DIS}}) = D'(G^{\text{DIS}}) - \beta(V^*)'(G^{\text{DIS}}).$$

Solving the last two equations for $(V^*)'(G^{\text{DIS}})$ and $B'(R^{\text{BAU}} + g^{\text{DIS}}) - C'(g^{\text{DIS}})$ one obtains in particular

$$B'(R^{\text{BAU}} + g^{\text{DIS}}) - C'(g^{\text{DIS}}) = \frac{D'(G^{\text{DIS}})}{1 - \beta\gamma}.$$

Using the functional forms of B , C , and D as well as $g^{\text{DIS}} = \gamma G^{\text{DIS}}$, this equation can be written as

$$-R^{\text{BAU}} - (1 - \gamma)(1 + c)G^{\text{DIS}} + z = \frac{dG^{\text{DIS}}}{1 - \beta\gamma},$$

from which we obtain (33).

Finally, it has to be shown that the greenhouse gas stock converges monotonically to G^{DIS} . Because of (21) and (50) it is sufficient to verify that $0 < h_G + \gamma < 1$ holds. Using (29) we obtain

$$h_G + \gamma = \frac{\gamma(1+c)}{1+c+d - \beta v_{GG}}.$$

It follows immediately from this expression, from $\gamma < 1$, and from $v_{GG} < 0$ that $0 < h_G + \gamma < 1$ is satisfied. This completes the proof of the theorem.

Proof of Lemma 7

Starting from the conjectures stated in (41) and

$$\hat{V}^*(G_-) = \hat{v}_1 + \hat{v}_G G_- + \frac{\hat{v}_{GG}}{2} (G_-)^2$$

one can proceed analogously to the proof of Theorem 2 to find that

$$\begin{aligned} \mathbf{g}^*(G_-, R_i) &= \frac{z - R_i - \hat{d}\mathcal{H}(G_-) + \beta[\hat{v}_G + \hat{v}_{GG}\mathcal{H}(G_-)]}{1 + c}, \\ \hat{h}_R &= -\frac{1}{1 + c}, \\ \mathbf{R}^*(G_-) &= R^{\text{BAU}}, \\ \mathcal{H}(G_-) &= \hat{h}_1 + (\hat{h}_G + \gamma)G_- - \frac{R^{\text{BAU}}}{1 + c}. \end{aligned}$$

It is not difficult to see that there is no structural difference between the situation analyzed in Theorem 2 and the present case. That is, all we need to do is replace the parameter d wherever it appears by \hat{d} . This completes the proof.

Proof of Theorem 3

We know from Lemma 7 that $\hat{c} = c$ implies that $\mathbf{R}^*(G_-) = R^{\text{BAU}}$. This result is independent of the value of \hat{d} . In order to prove Theorem 3 it is therefore sufficient to show that the maximization of aggregate welfare (4) subject to (2) and $R_{i,t} = R^{\text{BAU}}$ for all $i \in [0, 1]$ and all $t \geq 0$ leads to the same emission levels as the equilibrium under a benevolent authority described in Theorem 2.

The maximization of (4) subject to (2) and $R_{i,t} = R^{\text{BAU}}$ for all $i \in [0, 1]$ and all $t \geq 0$ is equivalent to the maximization of

$$\sum_{t=0}^{+\infty} \beta^t \int_0^1 [B(R^{\text{BAU}} + g_{i,t}) - C(g_{i,t}) - D(G_t)] di$$

subject to (2). Concavity of B and strict convexity of C imply that $g_{i,t} = g_t$ holds for all $i \in [0, 1]$. The necessary and sufficient first-order optimality conditions are

$$\begin{aligned} G_t &= g_t + \gamma G_{t-1}, \\ D'(G_t) + C'(g_t) - B'(R^{\text{BAU}} + g_t) &= \beta \gamma [C'(g_{t+1}) - B'(R^{\text{BAU}} + g_{t+1})], \\ \lim_{t \rightarrow +\infty} \beta^t [C'(g_t) - B'(R^{\text{BAU}} + g_t)] G_t &= 0. \end{aligned}$$

Using the specification of the functions B , C , and D , the first two of these conditions can be rewritten as

$$\beta \gamma (1 + c) G_{t+1} - [d + (1 + \beta \gamma^2)(1 + c)] G_t + \gamma (1 + c) G_{t-1} = (1 - \beta \gamma)(R^{\text{BAU}} - z).$$

Using the definition of R^{BAU} it is easy to see that the unique fixed point of this difference equation is G^{DIS} as defined in (33). The eigenvalues of the difference equation are

$$\lambda_{1,2} = \frac{(1 + \beta \gamma^2)(1 + c) + d \pm \sqrt{[(1 + \beta \gamma^2)(1 + c) + d]^2 - 4\beta \gamma^2(1 + c)^2}}{2\beta \gamma(1 + c)}.$$

One can show that the smaller one of these two eigenvalues coincides with $h_G + \gamma$ where h_G is specified in (29). We also know from the proof of Theorem 2 that this value is an element of the interval $(0, 1)$. Hence, it follows that the optimal state trajectory of the optimization problem under consideration satisfies

$$G_t = (h_G + \gamma)G_{t-1} + (1 - h_G - \gamma)G^{\text{DIS}}.$$

Since this difference equation coincides with the aggregate law of motion $G_t = \mathcal{H}(G_{t-1})$ from Theorem 2, the proof of the present theorem is complete.

Formulas for the case $\hat{c} \neq c$

The general procedure to compute the equilibrium remains the same as in Lemma 7, but the algebra becomes more cumbersome. Consider the example from Section 5.2 with $\hat{c} \neq c$. Eq. (39) implies that

$$\mathbf{g}^*(G_-, R_i) = \hat{h}_1 + \hat{h}_G G_- + \hat{h}_R R_i = \frac{z - R_i - \hat{d}\mathcal{H}(G_-) + \beta[\hat{v}_G + \hat{v}_{GG}\mathcal{H}(G_-)]}{1 + \hat{c}}, \quad (53)$$

such that

$$\hat{h}_R = -\frac{1}{1+\hat{c}}.$$

Eq. (34) therefore gives

$$\mathcal{G}(G_-, \tilde{R}) = \hat{h}_1 + (\hat{h}_G + \gamma)G_- - \frac{1}{1+\hat{c}}\tilde{R}. \quad (54)$$

Solving condition (35) we obtain

$$\mathbf{R}^*(G_-) = \frac{(1+\hat{c})[\hat{h}_1(c-\hat{c}) - (1-\beta\rho)k(1+\hat{c}) + \hat{c}z]}{c+\hat{c}^2} + \frac{(1+\hat{c})\hat{h}_G(c-\hat{c})}{c+\hat{c}^2}G_-. \quad (55)$$

Combining (36), (37), (54), and (55) it follows that

$$\begin{aligned} \mathcal{H}(G_-) &= \hat{h}_1 + (\hat{h}_G + \gamma)G_- - \frac{1}{1+\hat{c}}\mathbf{R}^*(G_-) \\ &= \frac{\hat{h}_1(\hat{c}+\hat{c}^2) + (1-\beta\rho)k(1+\hat{c}) - \hat{c}z}{c+\hat{c}^2} + \left(\gamma + \frac{\hat{c}+\hat{c}^2}{c+\hat{c}^2}\hat{h}_G\right)G_-. \end{aligned} \quad (56)$$

Substituting (56) into (53) and comparing the coefficients of G_- on both sides, we can solve for \hat{h}_1 and \hat{h}_G in terms of \hat{v}_G and \hat{v}_{GG} , which yields

$$\begin{aligned} \hat{h}_1 &= \frac{(c+\hat{c}^2)z + \hat{d}[\hat{c}z - (1+\hat{c})(1-\beta\rho)k] + \beta[(c+\hat{c}^2)\hat{v}_G + (k+\hat{c}k - \hat{c}z)\hat{v}_{GG}] - \beta^2(1+\hat{c})k\rho\hat{v}_{GG}}{(1+\hat{c})[c+\hat{c}(\hat{c}+\hat{d}-\beta\hat{v}_{GG})]}, \\ \hat{h}_G &= -\frac{(c+\hat{c}^2)\gamma(\hat{d}-\beta\hat{v}_{GG})}{(1+\hat{c})[c+\hat{c}(\hat{c}+\hat{d}-\beta\hat{v}_{GG})]}. \end{aligned}$$

Combining these two results with (55) and (56) we can express $\mathbf{R}^*(G_-)$, $\mathbf{g}^*(G_-, \mathbf{R}^*(G_-))$, and $\mathcal{H}(G_-)$ in terms of the variable G_- and the undetermined coefficients \hat{v}_G and \hat{v}_{GG} . If we substitute these expressions into Eq. (38) and compare the coefficients of equal powers of G_- on both sides of the equation, we obtain three non-linear equations for the coefficients \hat{v}_1 , \hat{v}_G , and \hat{v}_{GG} . Once we have solved these equations, we have determined an equilibrium under discretion, i.e., the authority's policy function \mathbf{g}^* and value function \hat{V}^* . One can then solve Eq. (40) to determine aggregate welfare in this equilibrium.

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